

# PLANT BIOMECHANICS



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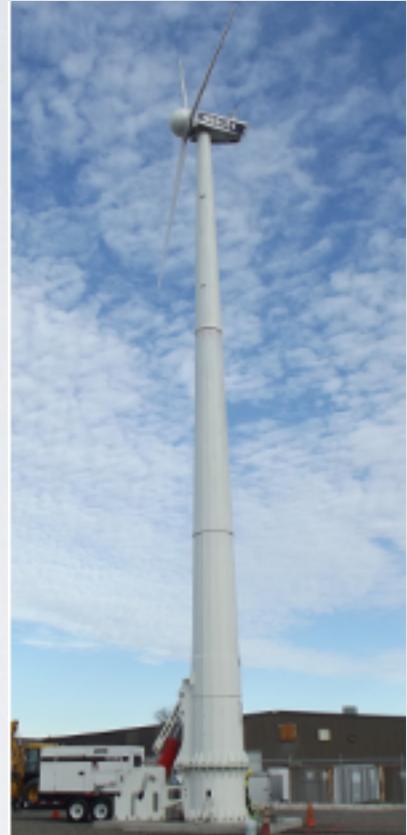
Web page: <http://loic.tadrist.pagesperso-orange.fr>

# EXAMPLES OF APPLIED PLANT BIOMIMETISM

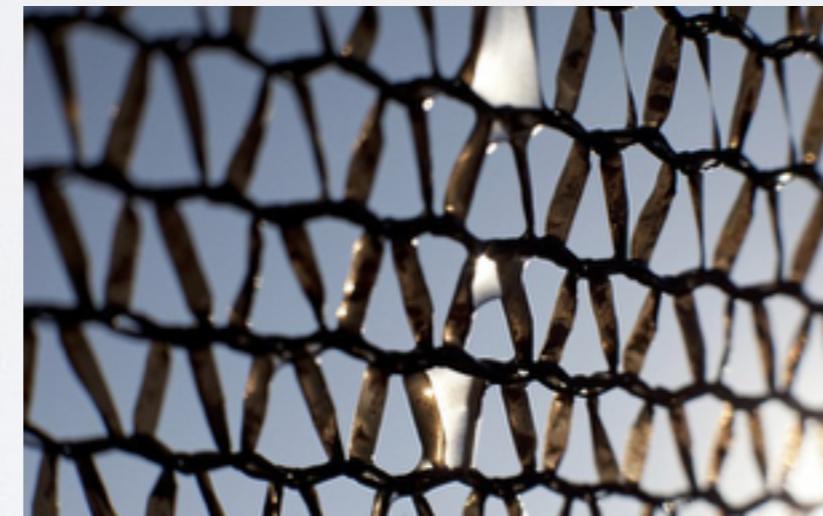
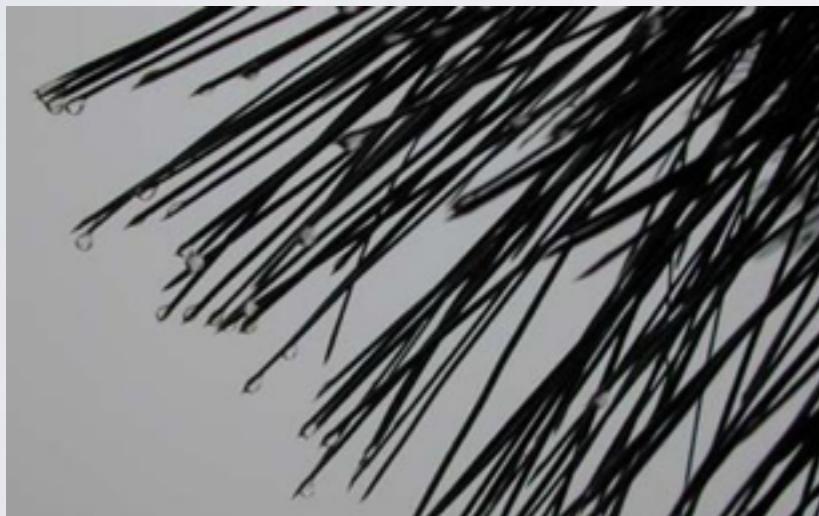
Self-cleaning surface



Poles in wind



Fog catching



Other examples: **architecture** inspired by plants, **water pumping**, resisting **negative pressures**, **hygroscopic motion**, **pressure dependant stiffness** material, etc.

... Also triggers questions of fundamental biology and evolution

# WHAT ARE THE PHYSICAL LIMITS FOR THE DESIGN OF A TREE TRUNK ?

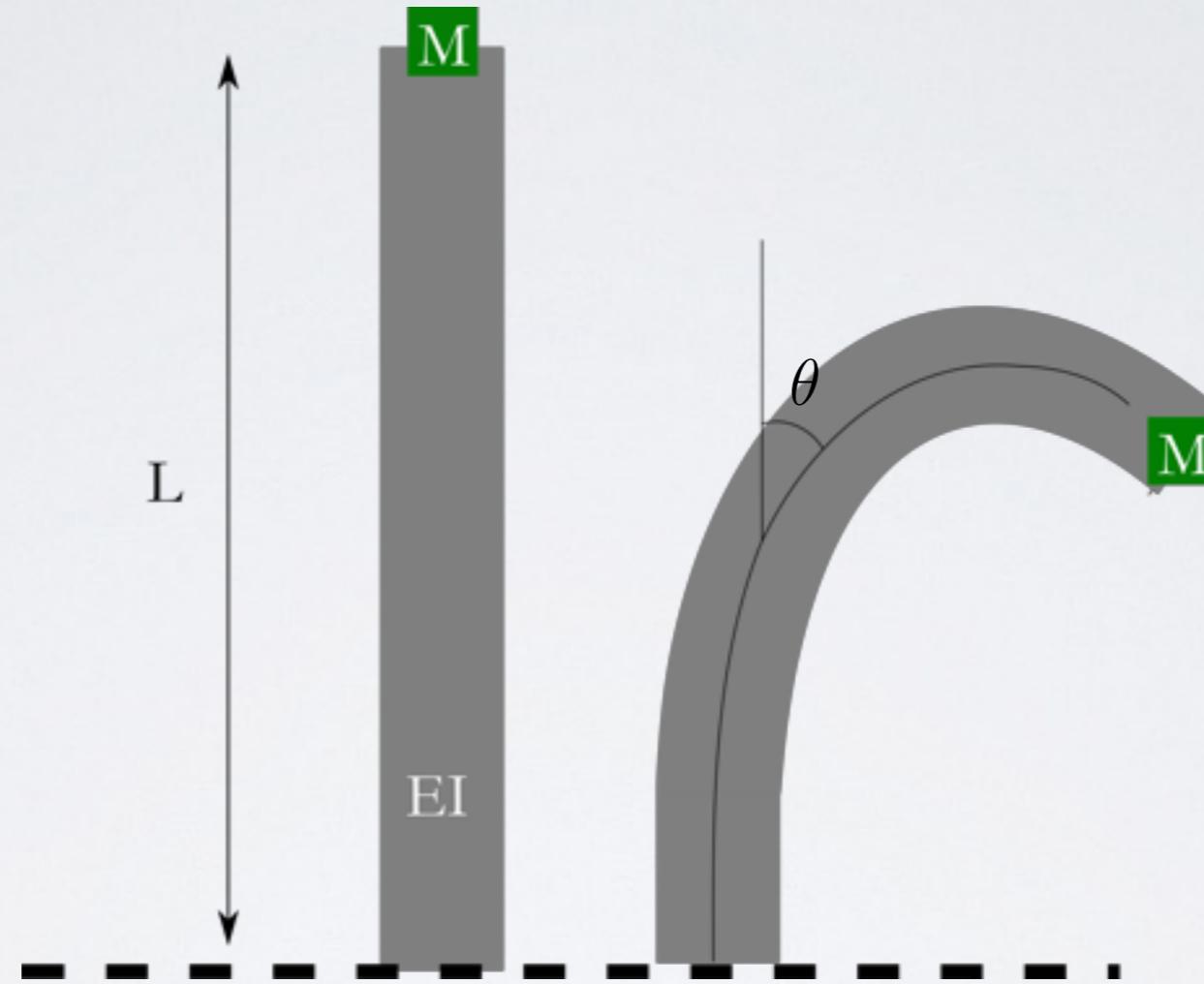


Race for light. Winner takes all.



Winner = Taller

# THE RISK OF BEING TALL: SELF BUCKLING



Beam equation:

$$\frac{d^2\theta}{ds^2} + \frac{Mg}{EI} \sin(\theta) = 0$$

Validity:

- Large displacement
- Small deformation

# THE RISK OF BEING TALL: SELF BUCKLING

Beam equation:

$$\frac{d^2\theta}{ds^2} + \frac{Mg}{EI} \sin(\theta) = 0$$

Validity:

- Large displacement
- Small deformation

Limits condition:

$$(d\theta/ds)_L = 0$$

No applied torque  
at free end

$$\theta(0) = 0$$

Clamping straight

Limit height\* for self buckling:

$$L \leq \sqrt{\frac{\pi^2 EI}{4Mg}}.$$

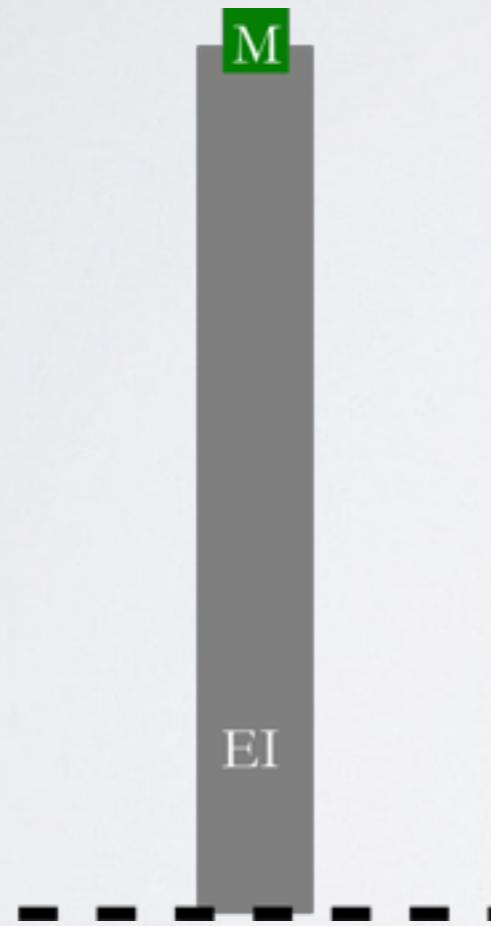
Euler's buckling condition

\*Maths are given in lecture notes



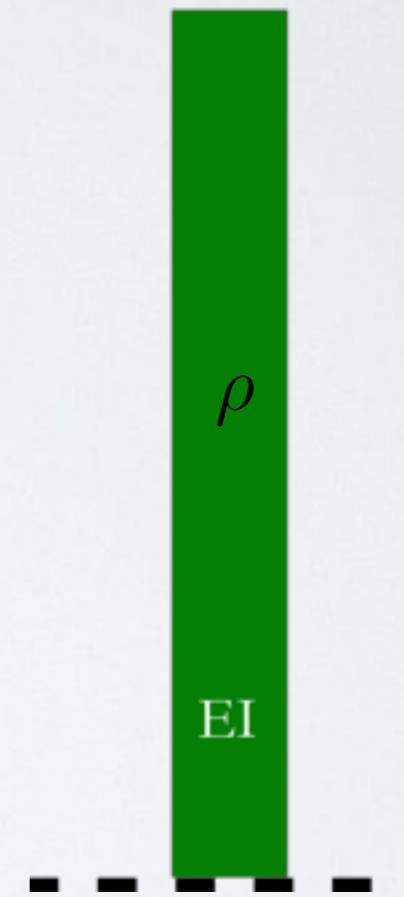
# THE RISK OF BEING TALL: SELF BUCKLING

Mass in the foliage

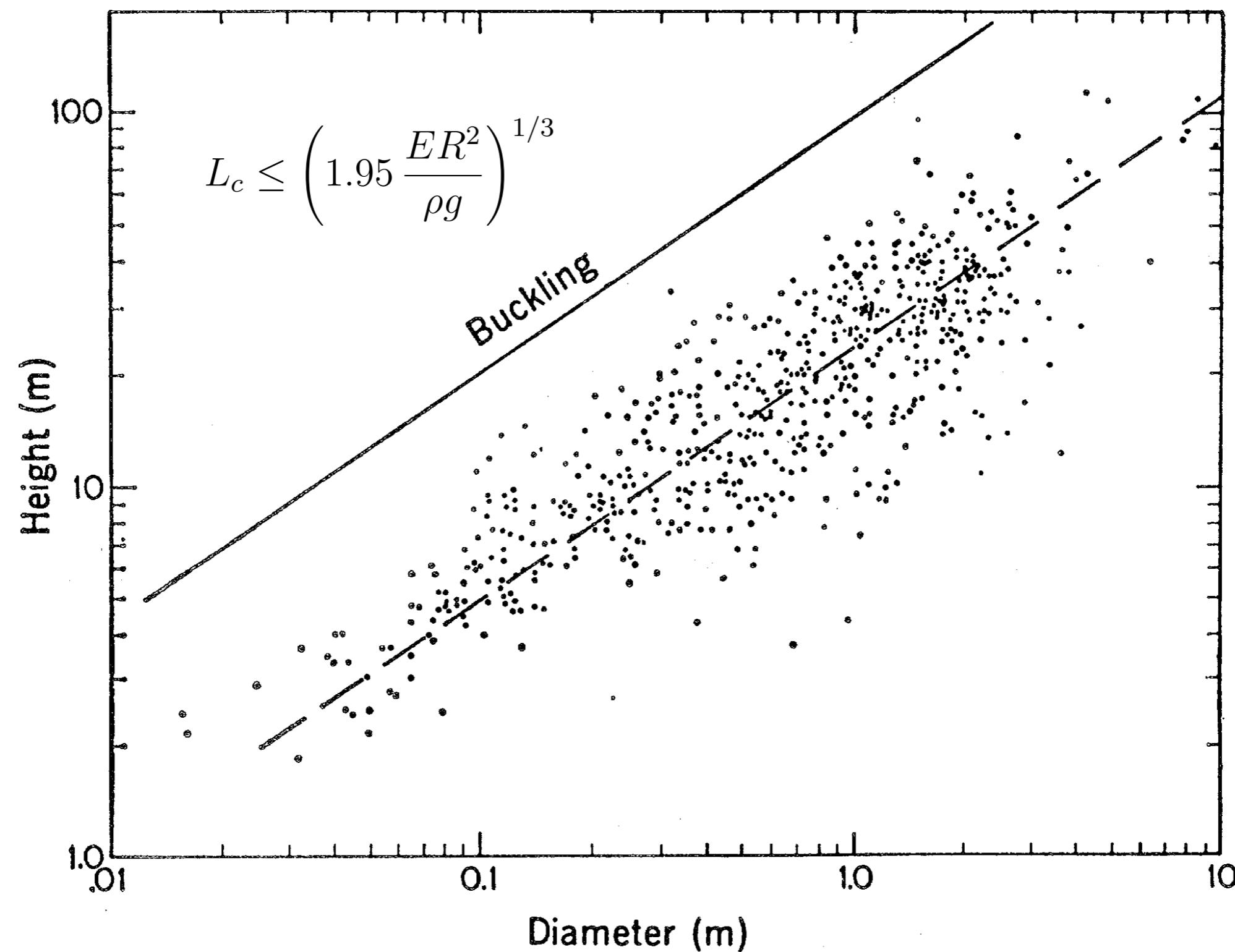


$$L \leq \sqrt{\frac{\pi^2 EI}{2Mg}}$$

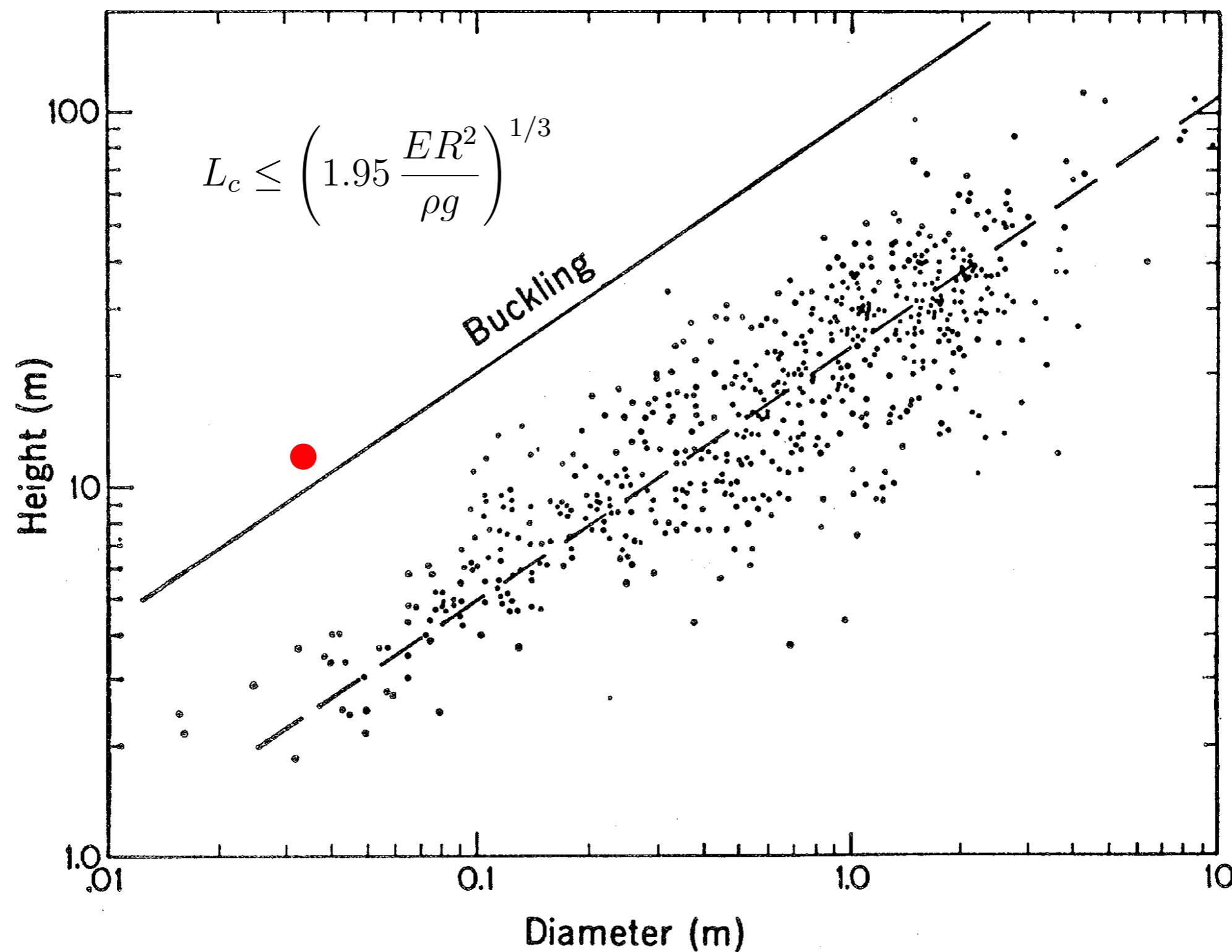
Mass in the trunk



$$L_c \leq \left( 1.95 \frac{ER^2}{\rho g} \right)^{1/3}$$



[McMahon, 1973]



[McMahon, 1973]

a



[Gardiner 2016]

# ARE PLANT MECHANICAL ENGINEERS?

	BOIS	ACIER	BÉTON
$\rho$ (kg.m <sup>-3</sup> )	450-1200	7500-8100	2200-2500
$E$ (GPa)	10-20	210	20-50
$R$ (m)	0.1	0.1	0.1

Can we build a wood tower as high as a concrete tower ?

Reminder:

$$L_c \leq \left( 1.95 \frac{ER^2}{\rho g} \right)^{1/3}$$

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$R$ (m)	0.1	0.1	0.1
$L_c$ (m)	31	37	27

Can we build a wood tower as high as a concrete tower ?

Even larger !

# MASS AND HEIGHT OF TREES

If a tree doubles the mass its trunk, how will change its height?

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If a tree double the massif its trunk how will change its height ?

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$$L \propto R^{2/3}$$

$$L \propto M^{1/4}$$

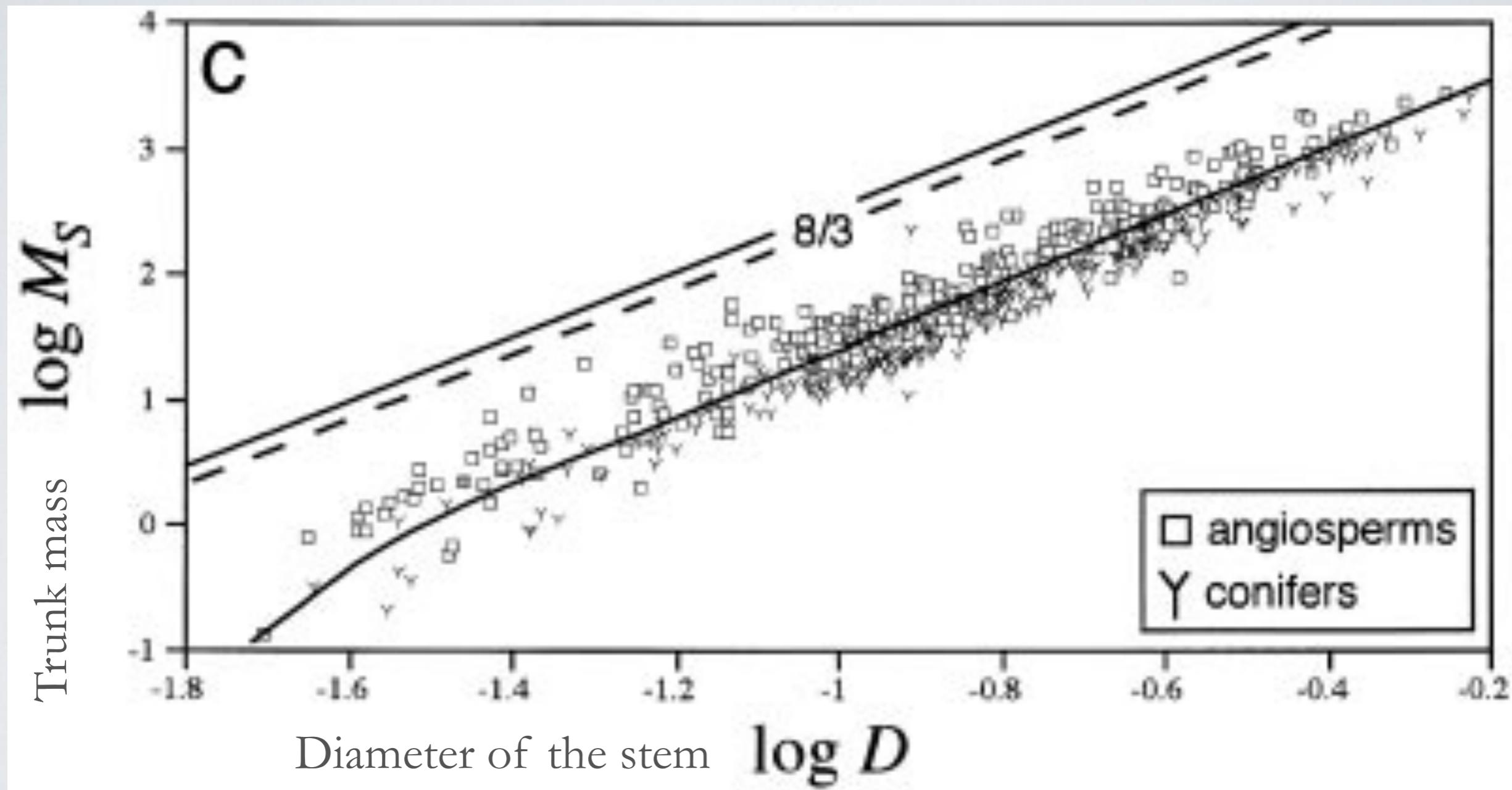
If the mass of the trunk doubles, the tree height only increases by 20%

$$M \propto LR^2$$

$$L \propto R^{2/3}$$



$$M \propto R^{8/3}$$



[Niklas & Spatz, 2004]

Is this tree shape optimised for self support ?

Trees are subjected to multiple environmental factors !



# RESISTING TO WIND: STAY STRONG OR BE FLEXIBLE



# Le Chêne et le Roseau

Le Chêne un jour dit au Roseau :  
" Vous avez bien sujet d'accuser la Nature ;  
Un Roitelet pour vous est un pesant fardeau.  
Le moindre vent, qui d'aventure  
Fait rider la face de l'eau,  
Vous oblige à baisser la tête :  
Cependant que mon front, au Caucase pareil,  
Non content d'arrêter les rayons du soleil,  
Brave l'effort de la tempête.  
Tout vous est Aquilon<sup>1</sup>, tout me semble Zéphyr<sup>2</sup>.  
Encor si vous naissiez à l'abri du feuillage  
Dont je couvre le voisinage,  
Vous n'auriez pas tant à souffrir :  
Je vous défendrais de l'orage ;  
Mais vous naissez le plus souvent  
Sur les humides bords des Royaumes du vent.  
La nature envers vous me semble bien injuste.  
- Votre compassion, lui répondit l'Arbuste,  
Part d'un bon naturel ; mais quittez ce souci.  
Les vents me sont moins qu'à vous redoutables.  
Je plie, et ne romps pas. Vous avez jusqu'ici  
Contre leurs coups épouvantables  
Résisté sans courber le dos ;  
Mais attendons la fin. " Comme il disait ces mots,  
Du bout de l'horizon accourt avec furie  
Le plus terrible des enfants  
Que le Nord eût portés jusque-là dans ses flancs.  
**L'Arbre tient bon ; le Roseau plie.**  
**Le vent redouble ses efforts,**  
**Et fait si bien qu'il déracine**  
**Celui de qui la tête au Ciel était voisine**  
**Et dont les pieds touchaient à l'Empire des Morts.**

Jean de la Fontaine  
Les Fables

# AGROFORESTRY CHALLENGES



Tree breaking



Tree uprooting

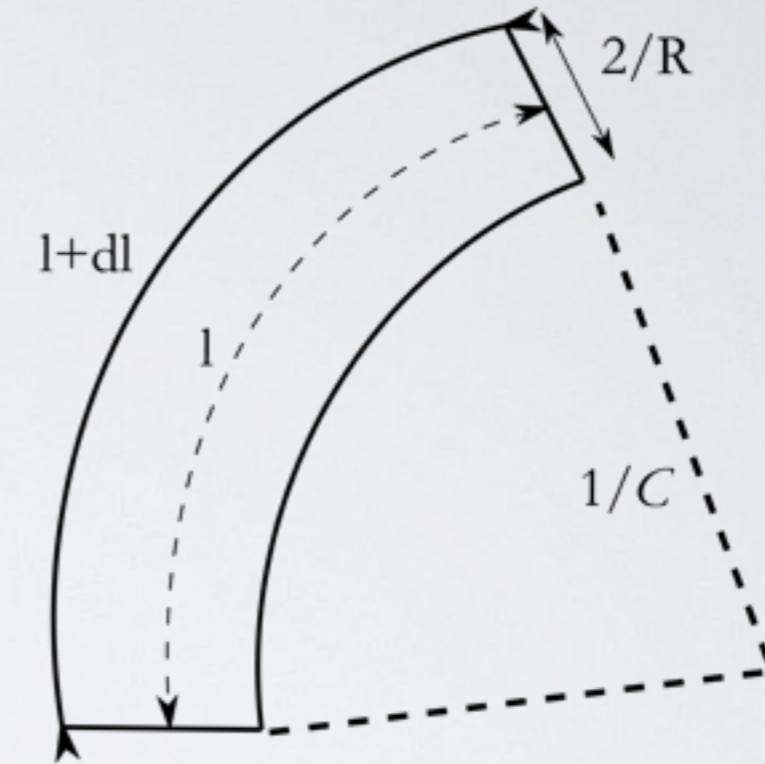
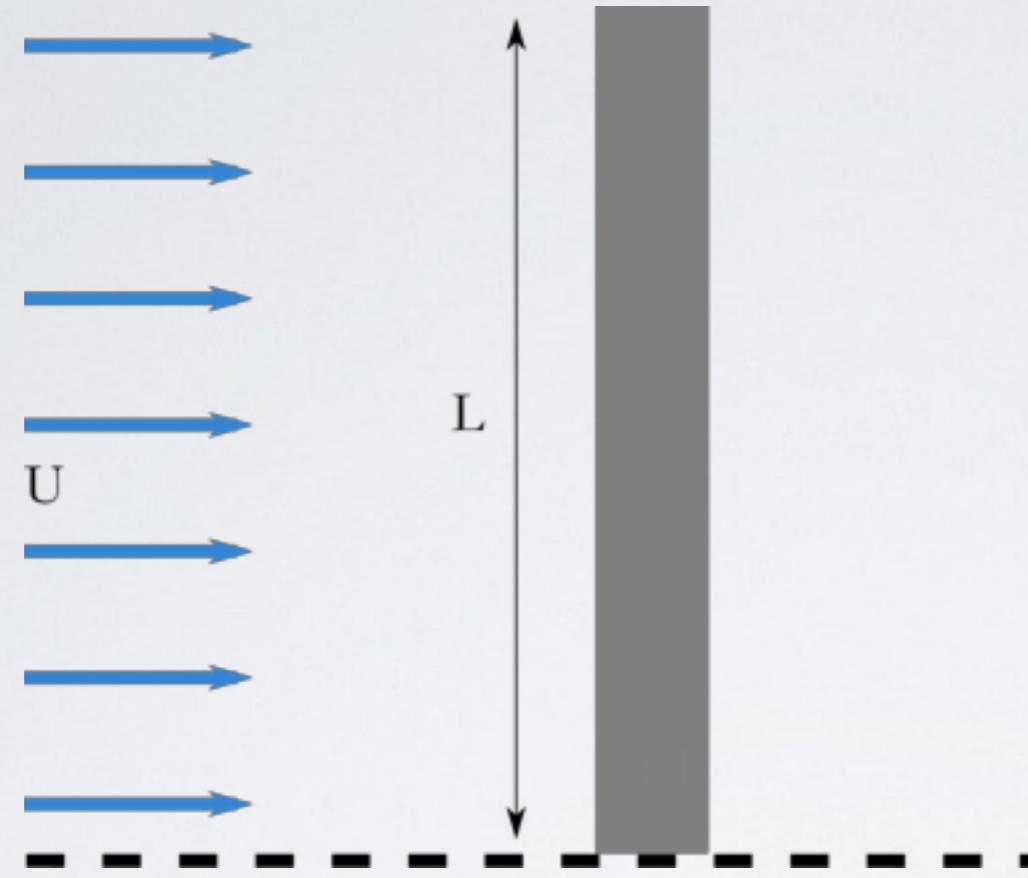


Massive losses in tree farming

France 1999:

- 140 000 000 m<sup>3</sup> of wood destroyed
- 5 billiard € of losses

# TRUNK DESIGN: AVOID TREE BREAKAGE



Torques equilibrium:

$$M = EI(s) \frac{d\theta}{ds} = F(L - s)$$

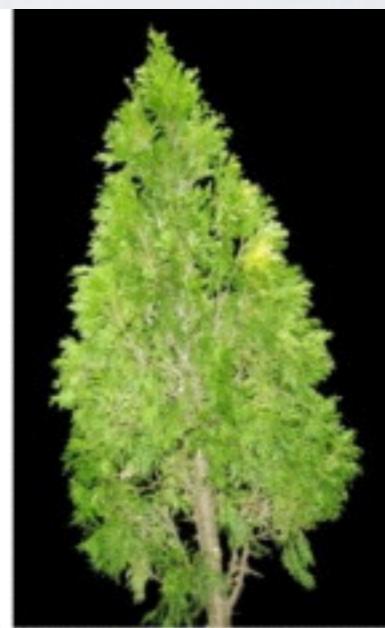
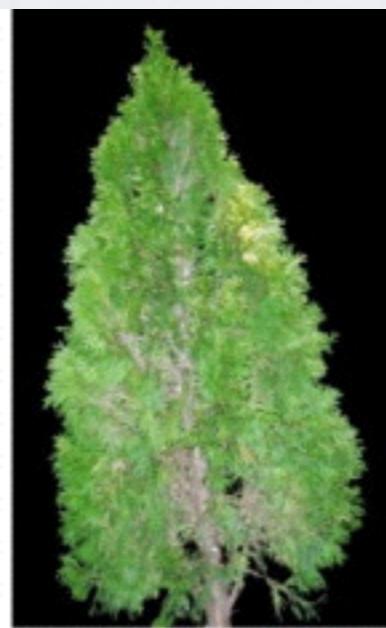
Maximal stress:

$$\sigma \propto \frac{RF(L - s)}{I(s)} = \frac{4F(L - s)}{\pi R(s)^3}$$

Trunk design:

$$R \propto (L - s)^{1/3}$$

(Constant maximal stress)



# TRUNK DESIGN: BRITTLE RECONFIGURATION



[Lopez, 2011]

Brittle reconfiguration:

Leaves and twigs break before large branches. This reduces the aerodynamic load and preserve the large limbs.

- Leonardo da Vinci's rule:  
“all the branches of a tree at every stage of its height when put together are equal in thickness to the trunk” [Eloy, 2011]

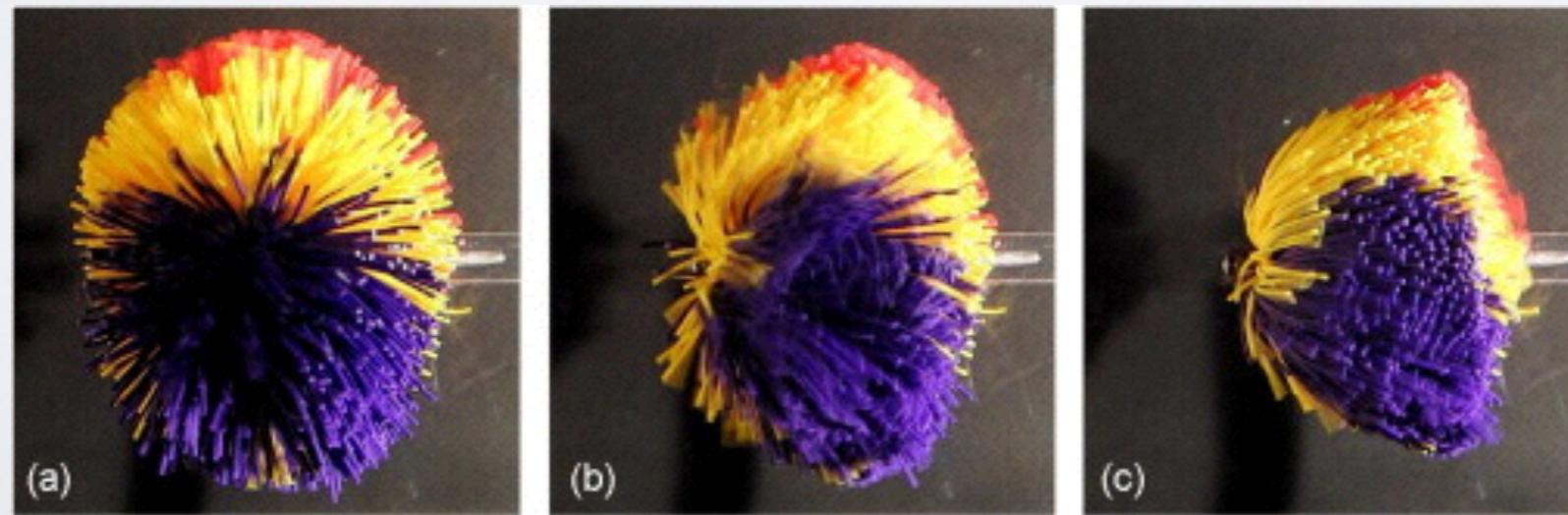
Separation in two branches (allometry 1/2)

$$R_f = R/2 \quad L_f = L/2 \quad F_f = F/2 \quad \rightarrow \quad \sigma_f = 2\sigma$$

$$\sigma \propto \frac{RF(L-s)}{I(s)} = \frac{4F(L-s)}{\pi R(s)^3}$$

Mother branch  
protected

# TRUNK DESIGN: ELASTIC RECONFIGURATION

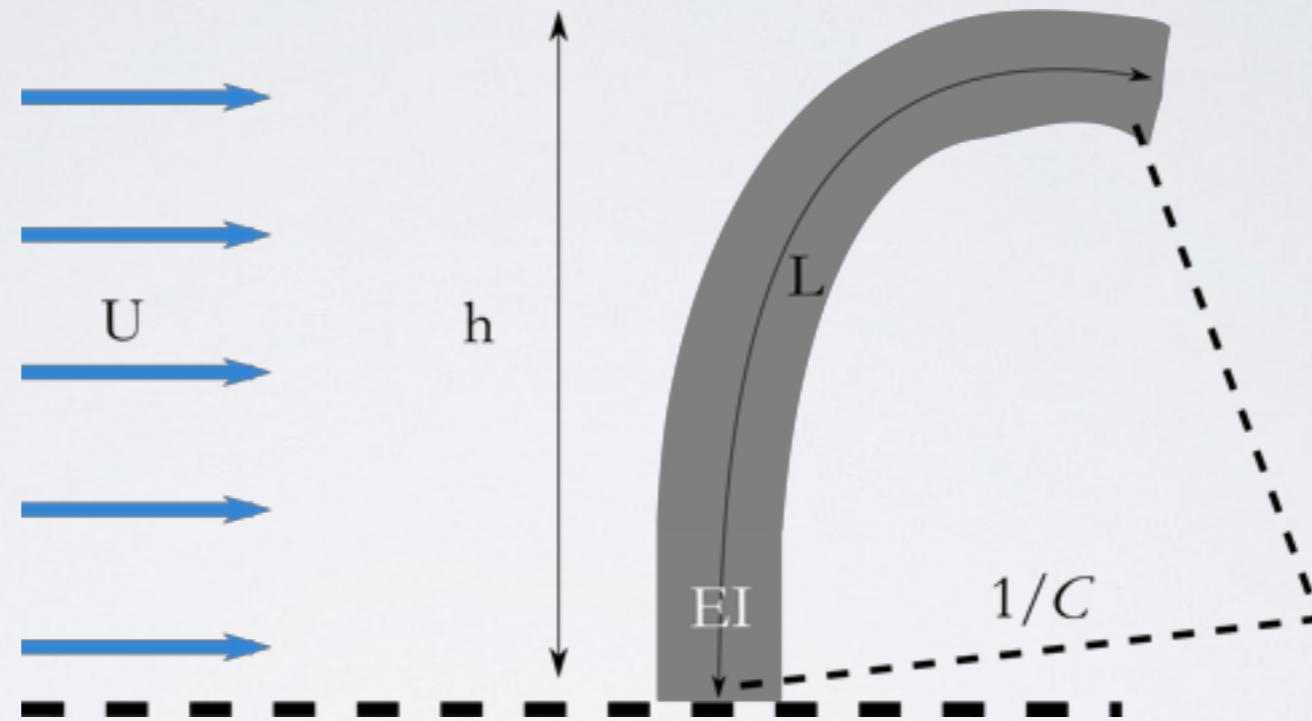


[Gosselin, 2009]

# TRUNK DESIGN: ELASTIC RECONFIGURATION

Drag of a solid:

$$F = \rho U^2 h w$$



Elastic deformation equilibrium:

$$EI\mathcal{C} = Fh \iff \left(\frac{h}{L}\right)^3 \propto \frac{EI}{\rho U^2 w L^3} = C_Y^{-1}$$

Cauchy number

$$C_Y = \frac{\text{Fluid force}}{\text{Elastic force}}$$

Drag of an elastic object :

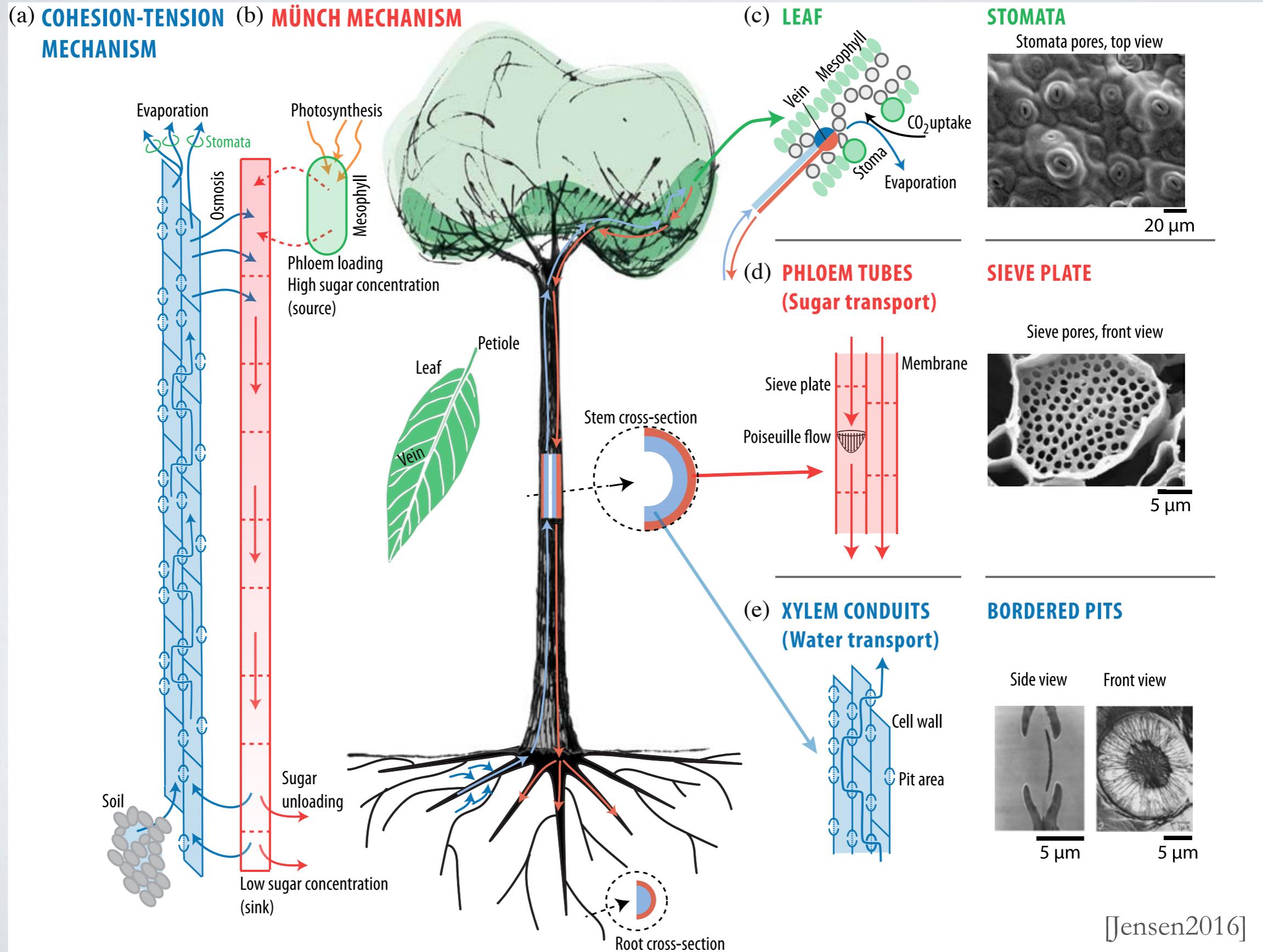
$$F \propto U^{2-\nu} \quad \text{with} \quad \nu = -\frac{2}{3}$$

Vogel coefficient

# TRUNK DESIGN: OTHER MODES OF PROTECTION WHEN IN WINDY CONDITIONS

- Energy transmission to small scales: non linear inertial transmission
- Energy transmission by branch to branch impacts

# ARE PLANT GOOD PLUMBERS?



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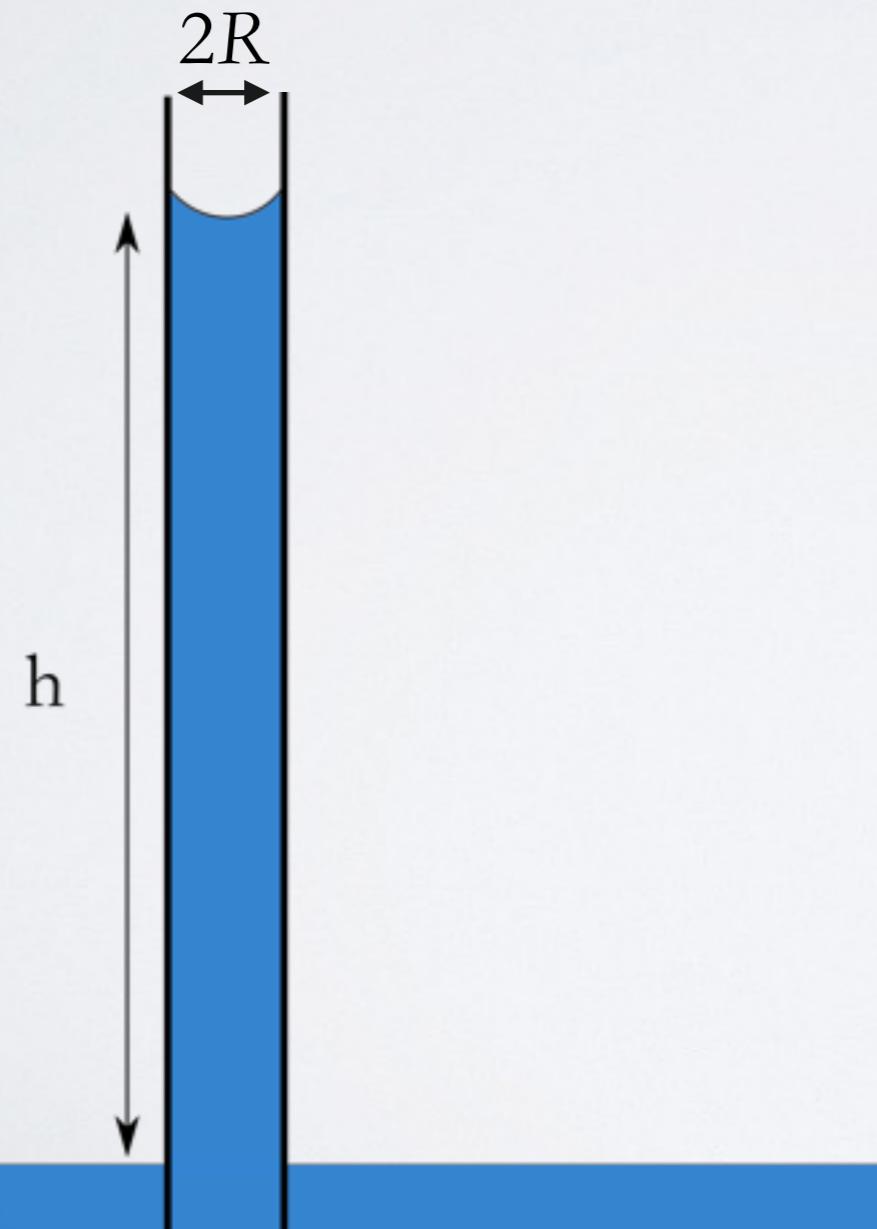
## 1. RISING WATER

- What is the maximal height you can expect a tree may rise water ? [Maximal tree size : 115 m, cells size : 10 microns]
- Can pressure be negative ?
- How much can tree resist negative pressure ?
- What is the physical limit to large trees ?

# ARE PLANT GOOD PLUMBERS?

## 1. RISING WATER

- what is the maximal height you can expect a tree may rise water ? [Maximal tree size : 115 m, cells size : 10 microns]



$R$  is around cell size

$2\gamma/R$  Laplace pressure difference

$\rho gh$  Hydrostatic pressure difference

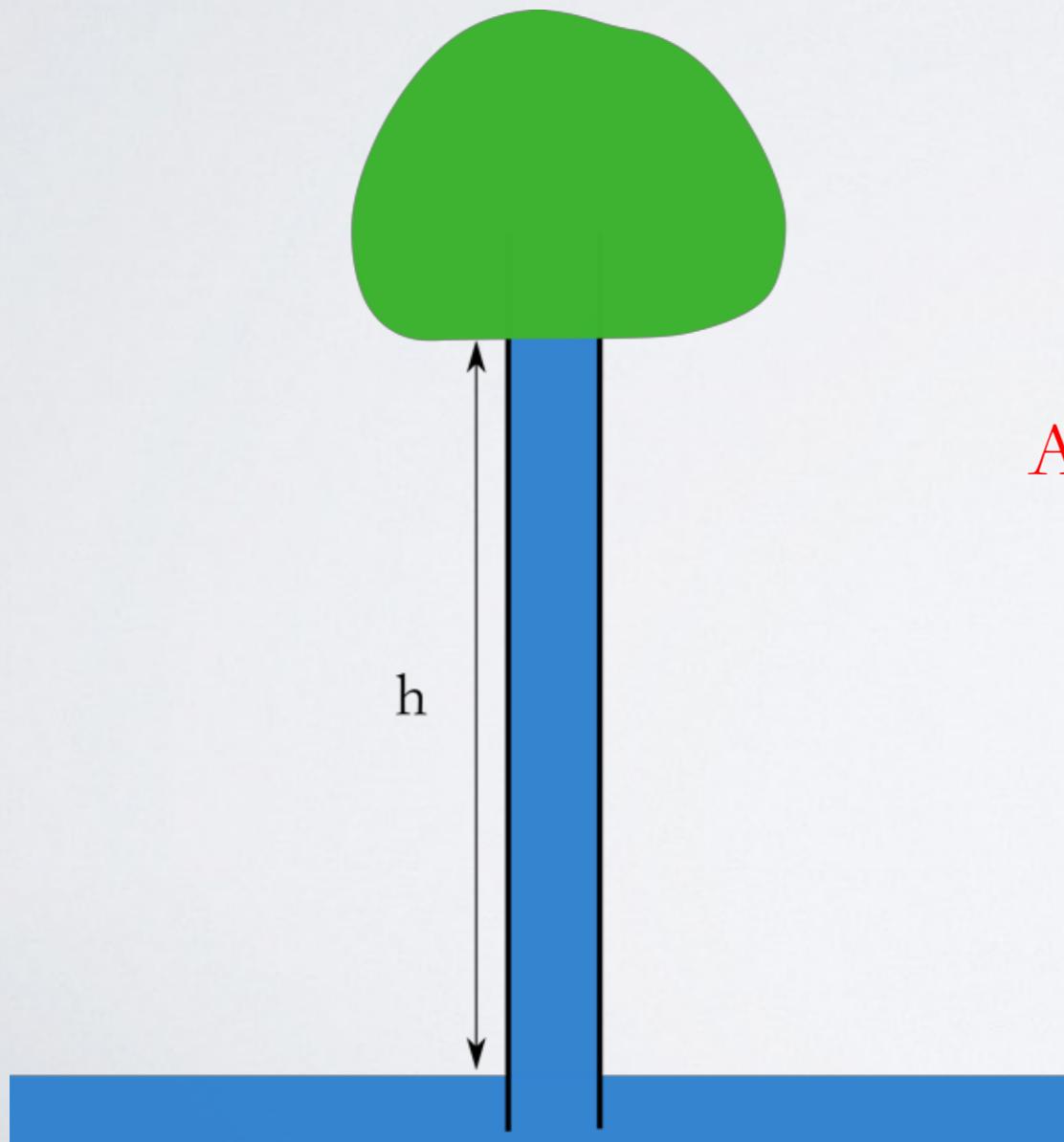
$$h = \frac{2\gamma}{\rho g R}$$

Only 3m !

# ARE PLANTS GOOD PLUMBERS?

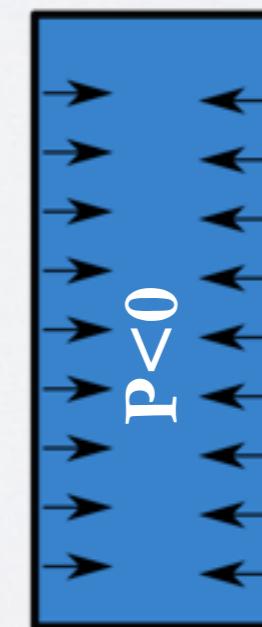
## 1. RISING WATER

- Can pressure be negative ? [Maximal tree size : 115 m]



$$P - P_0 = \rho g h$$

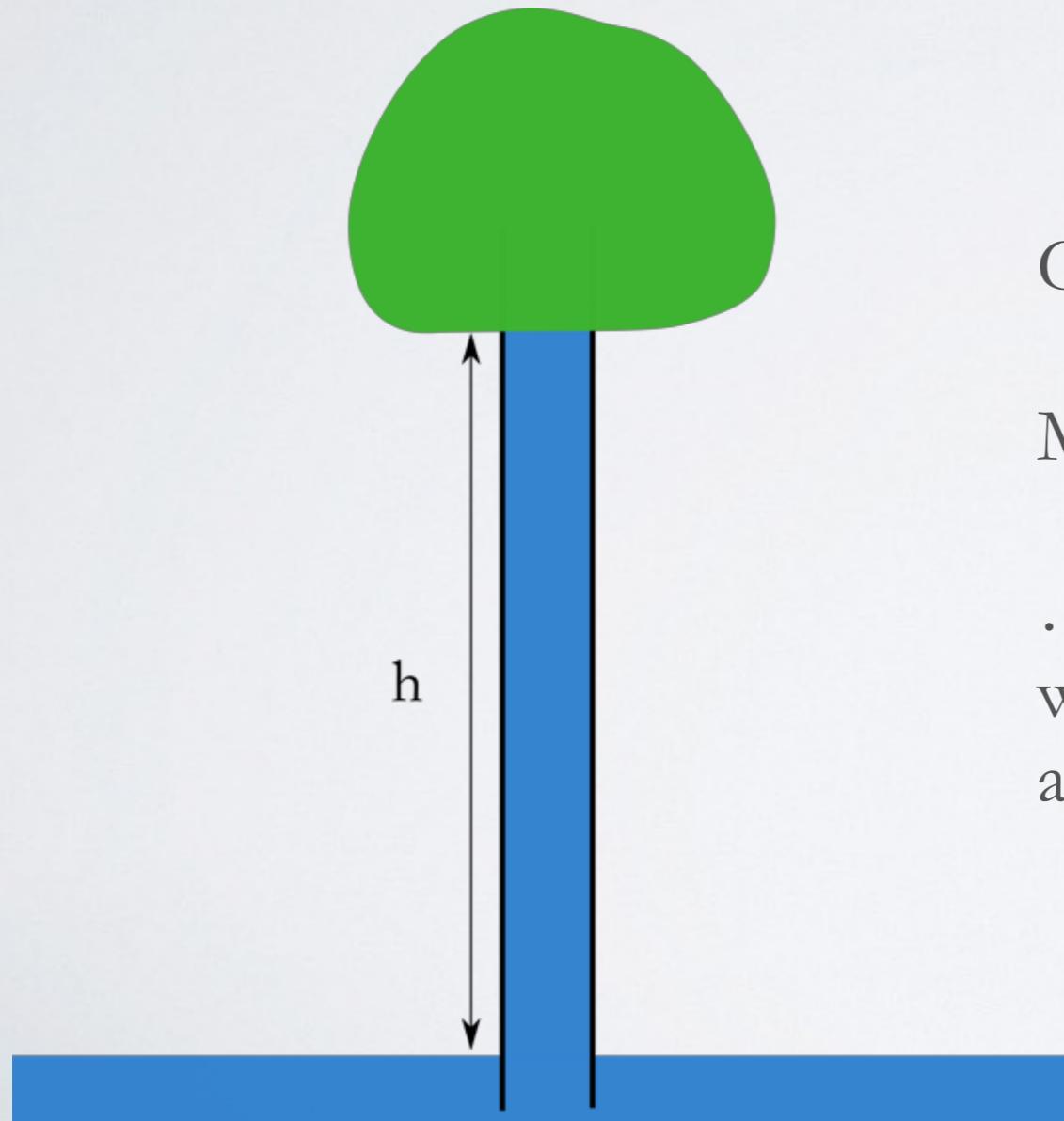
At top minus 10 times atmospheric pressure !



# ARE PLANT GOOD PLUMBERS?

## 1. RISING WATER

- What does bring water on top of a tree ?



Chemical potential of water !

Might be able to bring water up to 10000 m.

...but pressure would be so negative that it would cost a lot in terms of energy to build and renew such cells.

# ARE PLANT GOOD PLUMBERS?

## 2. STRENGTH OF THE TUBES

How much trees can resist negative pressure ?

For plants with a rigid cell wall:

Maximum pressure difference between in/out cell (thanks to rigid wall) :

$$\Delta P = 10^6$$

For animals without the rigid wall :

$$\Delta P = 10^4$$

Can you tell what is the maximal size of trees?

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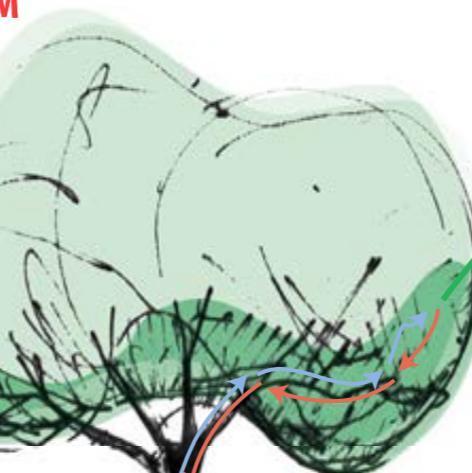
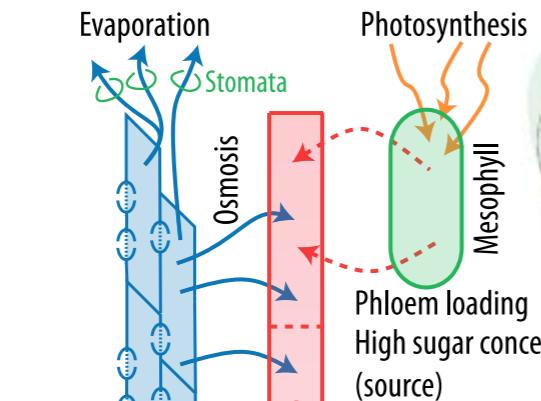
Maximal size of trees is ruled by many factors:

1. Mass-Height relationship (very slow convergence)
2. The energetic cost to bring water on top (xylem tubes reinforcement)
3. Resistance to drought and embolism recovery
4. Fight against biotic and abiotic factors
5. Maintenance of photosynthetic yield
6. Others

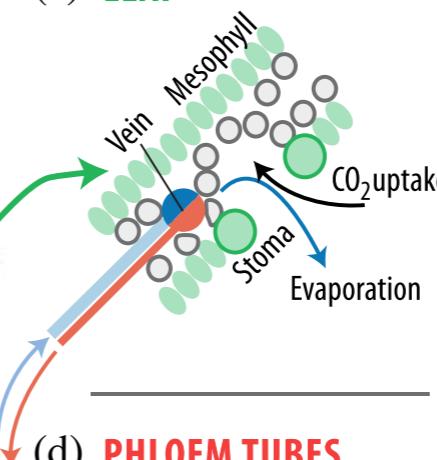
Maximum size of trees is estimated between 110 - 140 m

# ARE PLANT GOOD PLUMBERS?

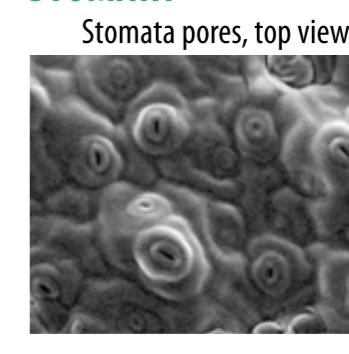
(a) **COHESION-TENSION MECHANISM**



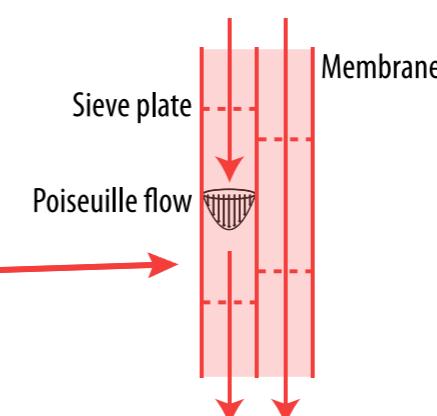
(c) **LEAF**



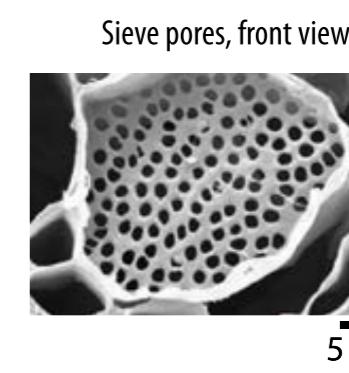
**STOMATA**



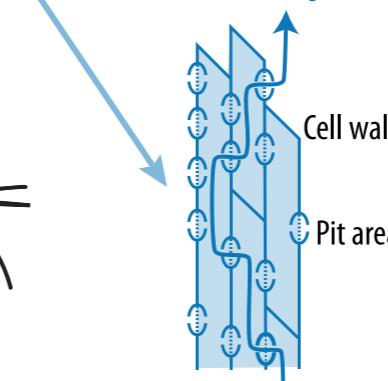
(d) **PHLOEM TUBES (Sugar transport)**



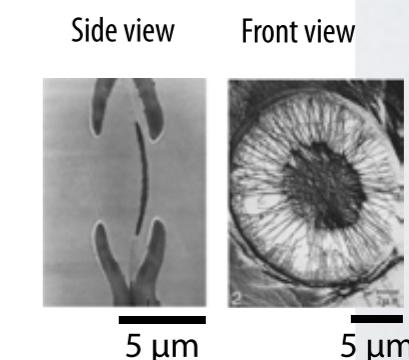
**SIEVE PLATE**



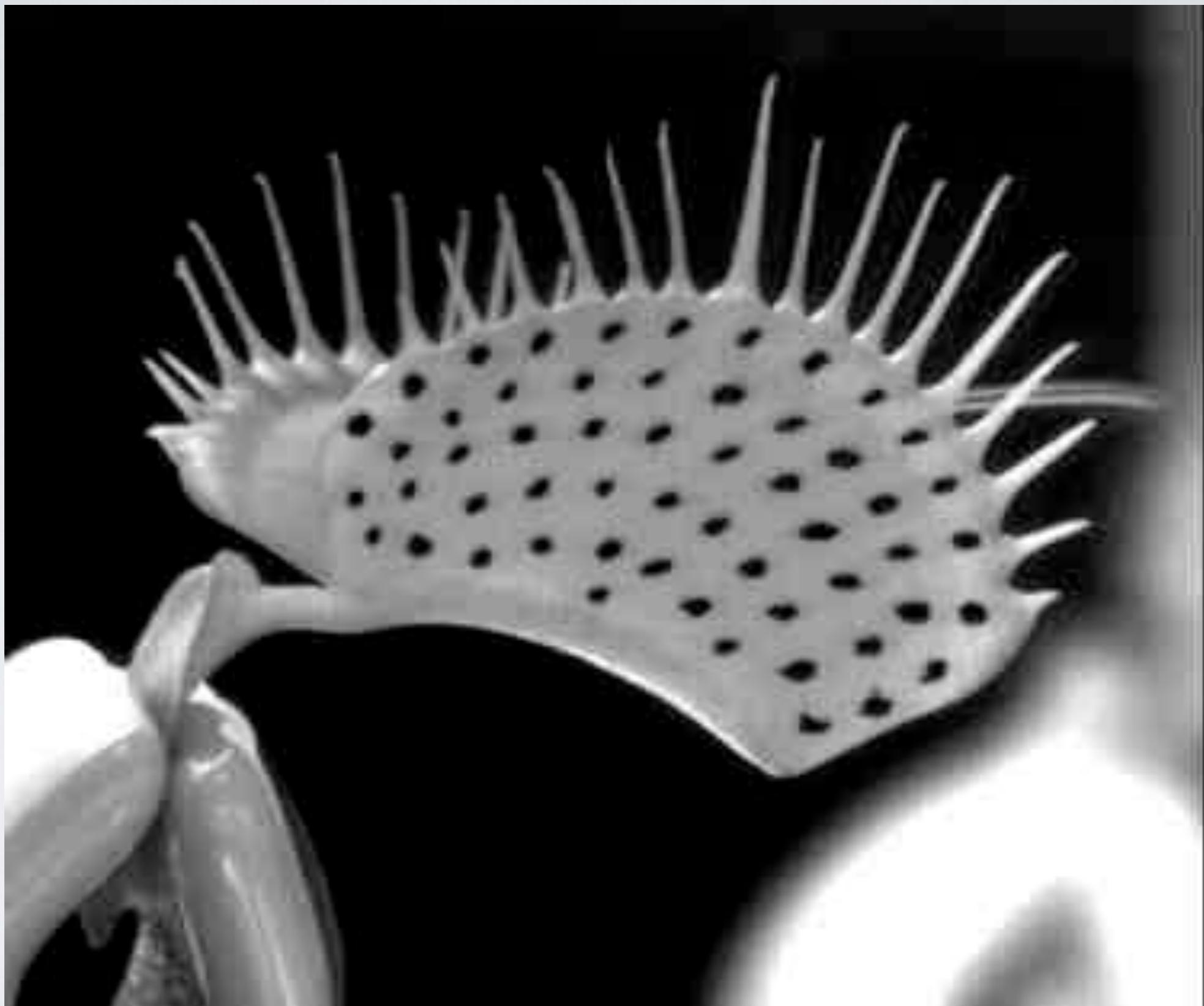
(e) **XYLEM CONDUITS (Water transport)**



**BORDERED PITS**



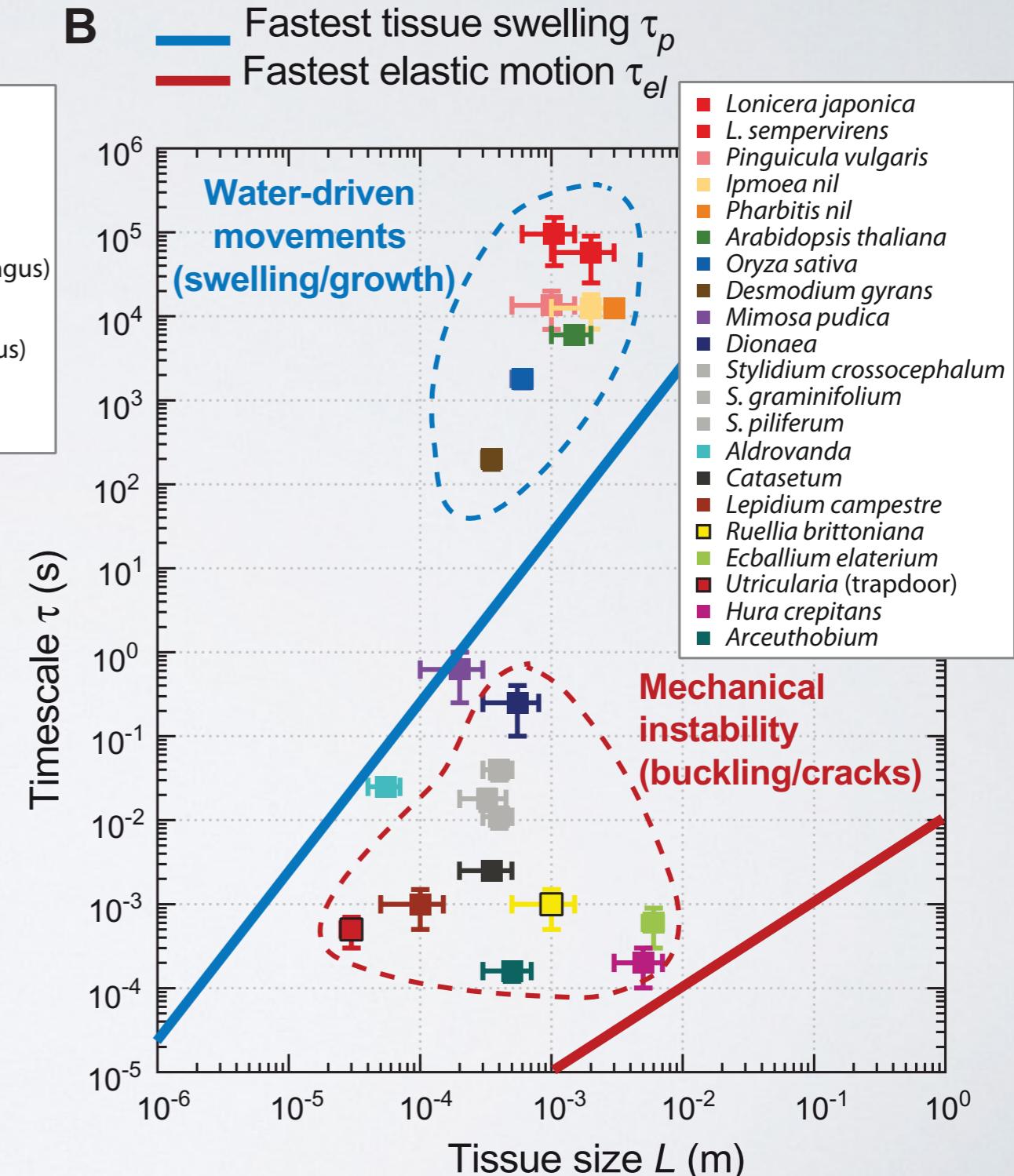
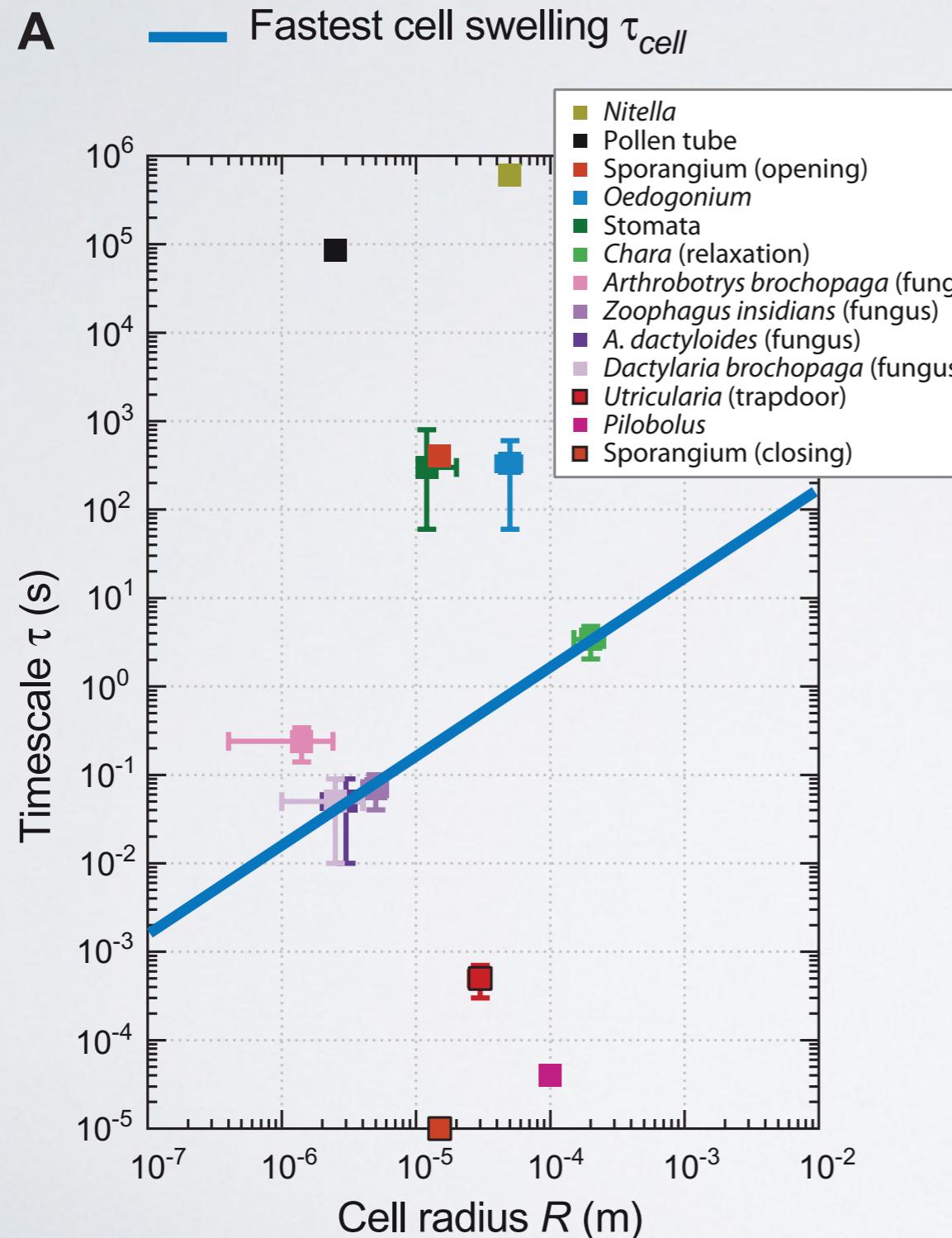
# PLANT FAST MOTIONS: HOW TO MOVE WITHOUT MUSCLES?



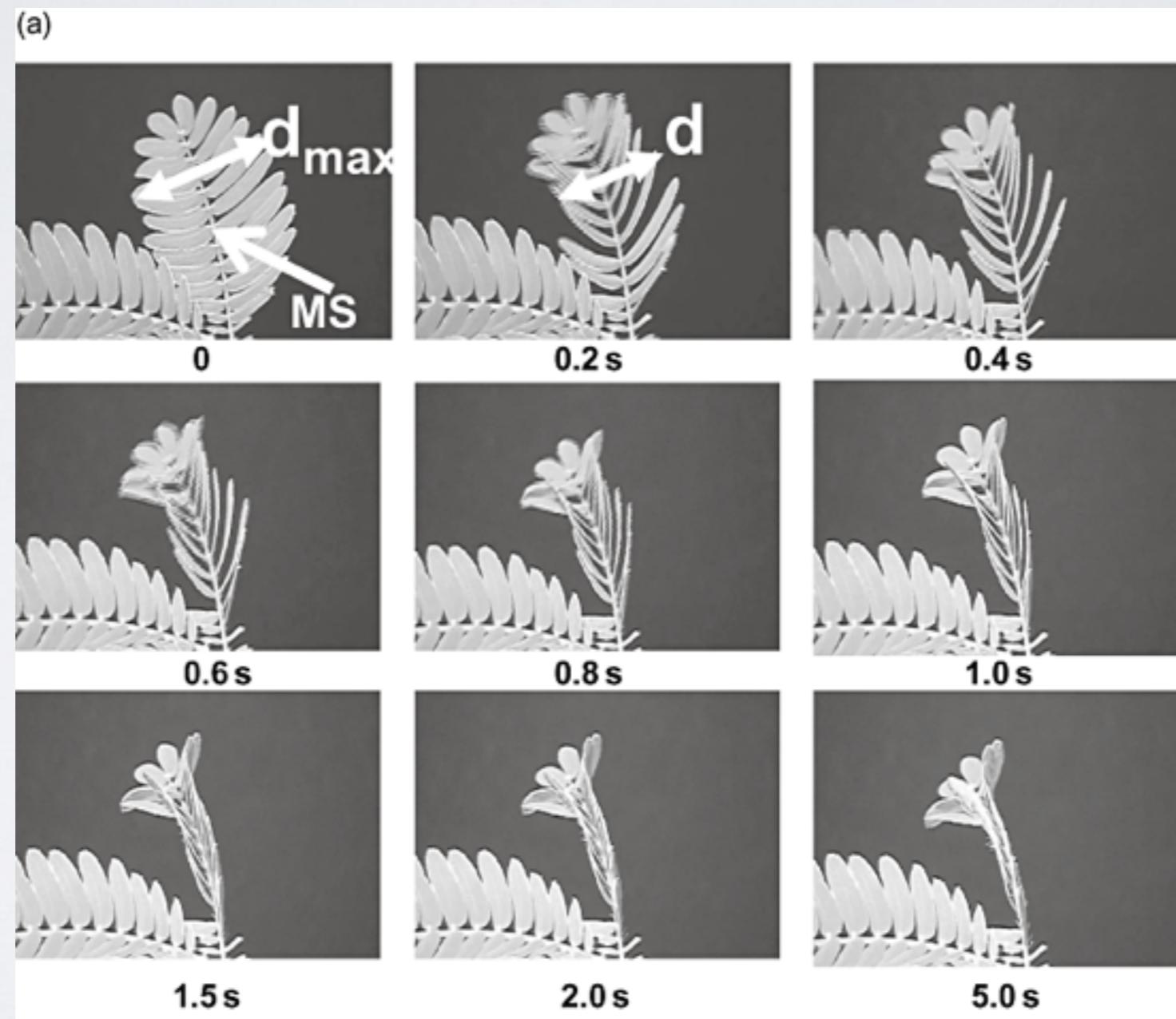
Slowed down 30 times

[Forterre 2005]

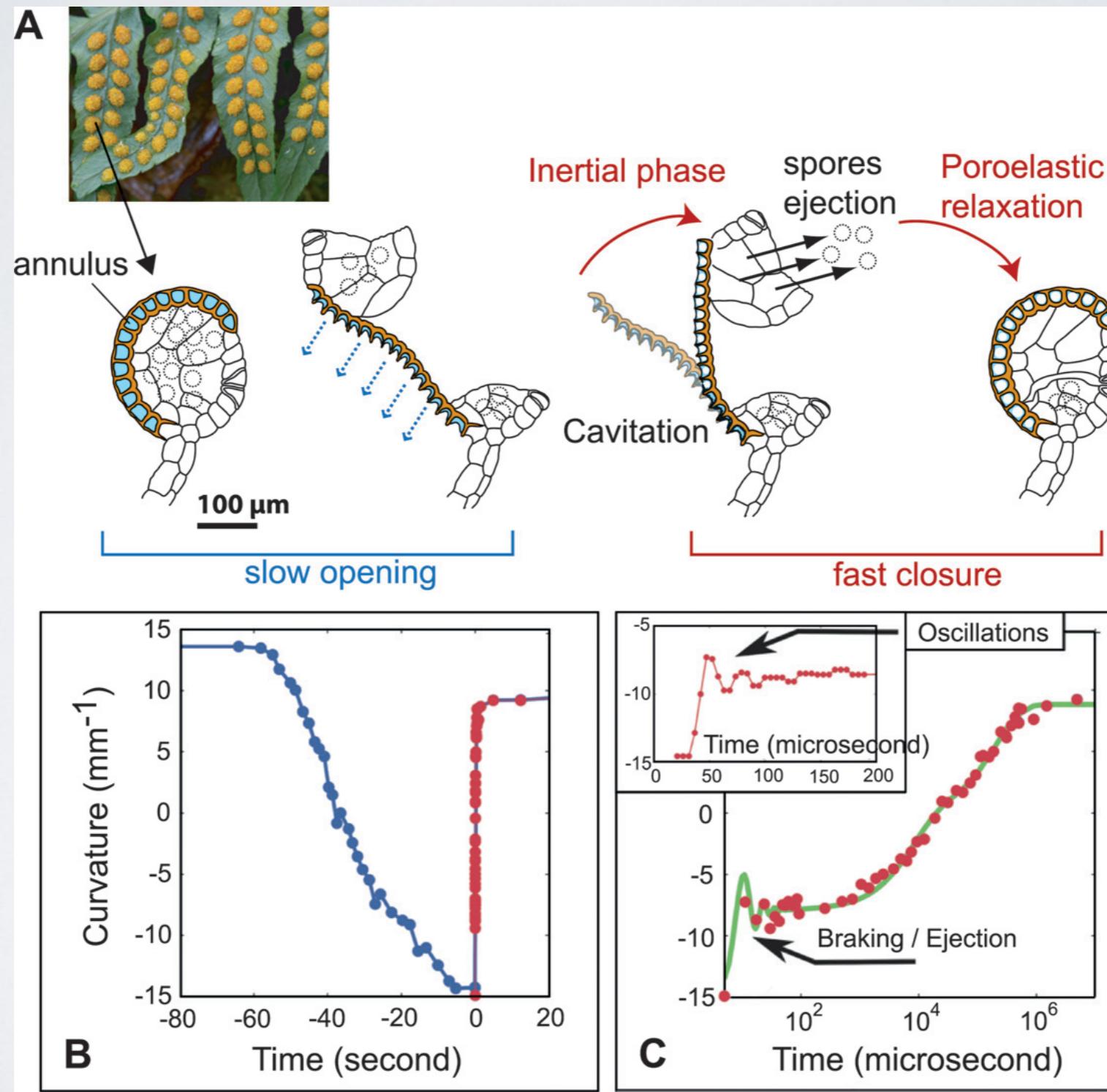
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[Noblin 2016]

# OTHER FIELDS OF PLANT BIOMECHANICS

- Growth
- Leaf motion in wind (application in pesticides spreading and in the making of virtual scenes)
- Splash of rain on leaves (disease spreading)
- Fruit dispersal in wind (ex: dandelion)
- Shape of spines
- Rheological properties of some Carnivorous plant with liquid traps
- Fluid transport optimality
- [...]

## Key message :

### **Optimality in nature responds to multiple factors**

Abiotic factors: Light collection, mechanical support, wind resistance, fluid transport...  
Biotic factors: response to herbivory, attacks by microbes, ...

### **Optimality for humans concerns one or two factors**

Fog catching net are only devoted to catch water not also to collect light

True biomimetism is rare I prefer talking about bio-inspiration

Plants biomechanics explore much more wonders ...



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